

Correlations between SIDIS azimuthal asymmetries in target and current fragmentation regions

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Summary. — We shortly describe the leading twist formalism for spin and transverse-momentum dependent fracture functions recently developed and present results for the production of spinless hadrons in the target fragmentation region (TFR) of SIDIS [1]. In this case not all fracture functions can be accessed and only a Sivers-like single spin azimuthal asymmetry shows up at LO cross-section. Then, we show [2] that the process of double hadron production in polarized SIDIS – with one spinless hadron produced in the current fragmentation region (CFR) and another in the TFR – would provide access to all 16 leading twist fracture functions. Some particular cases are presented.

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1. – Introduction

As it is becoming increasingly clear in the last decades, the study of the three-dimensional spin-dependent partonic structure of the nucleon in SIDIS processes requires a full understanding of the hadronization process after the hard lepton-quark scattering. So far most SIDIS experiments were studied in the CFR, where an adequate theoretical formalism based on distribution and fragmentation functions has been established (see for example Ref. [3]). However, to avoid misinterpretations, also the factorized approach to SIDIS description in the TFR has to be explored. The corresponding theoretical basis – the fracture functions formalism – was established in Ref. [4] for hadron transverse momentum integrated unpolarized cross-section. Recently this approach was generalized [1] to the spin and transverse momentum dependent case (STMD).

We consider the process (adopting the same notations as in Ref. [2])

$$(1) \quad l(\ell, \lambda) + N(P, S) \rightarrow l(\ell') + h(P_h) + X(P_X)$$

with the hadron h produced in the TFR. We use the standard DIS notations and in the $\gamma^* - N$ c.m. frame we define the z -axis along the direction of \mathbf{q} (the virtual photon momentum) and the x -axis along ℓ_T , the lepton transverse momentum. The kinematics of the produced hadron is defined by the variable $\zeta = P_h^-/P^- \simeq E_h/E$ and its transverse momentum $\mathbf{P}_{h\perp}$ (with magnitude $P_{h\perp}$ and azimuthal angle ϕ_h). Assuming TMD factorization the cross-section of the process (1) can be written as

$$(2) \quad \frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\rightarrow\ell(\ell')+h(P)+X}}{dx_B dQ^2 d\zeta d^2\mathbf{P}_{h\perp} d\phi_S} = \mathcal{M} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\rightarrow\ell(\ell')+q(k',s')}}{dQ^2},$$

where ϕ_S is the azimuthal angle of the nucleon transverse polarization. The STMD fracture functions \mathcal{M} has a clear probabilistic meaning: it is the conditional probability to produce a hadron h in the TFR when the hard scattering occurs on a quark q from the target nucleon N . The expression of the non-coplanar polarized lepton-quark hard scattering cross-section can be found in Ref. [5].

The most general expression of the LO STMD fracture functions for unpolarized ($\mathcal{M}^{[\gamma^-]}$), longitudinally polarized ($\mathcal{M}^{[\gamma^- \gamma_5]}$) and transversely polarized ($\mathcal{M}^{[i \sigma^{i-} \gamma_5]}$) quarks are introduced in the expansion of the leading twist projections as [1, 2]:

$$(3) \quad \mathcal{M}^{[\gamma^-]} = \hat{u}_1 + \frac{\mathbf{P}_{h\perp} \times \mathbf{S}_\perp}{m_h} \hat{u}_{1T}^h + \frac{\mathbf{k}_\perp \times \mathbf{S}_\perp}{m_N} \hat{u}_{1T}^\perp + \frac{S_\parallel (\mathbf{k}_\perp \times \mathbf{P}_{h\perp})}{m_N m_h} \hat{u}_{1L}^\perp$$

$$(4) \quad \mathcal{M}^{[\gamma^- \gamma_5]} = S_\parallel \hat{l}_{1L} + \frac{\mathbf{P}_{h\perp} \cdot \mathbf{S}_\perp}{m_h} \hat{l}_{1T}^h + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{m_N} \hat{l}_{1T}^\perp + \frac{\mathbf{k}_\perp \times \mathbf{P}_{h\perp}}{m_N m_h} \hat{l}_1^\perp$$

$$(5) \quad \begin{aligned} \mathcal{M}^{[i \sigma^{i-} \gamma_5]} = & S_\perp^i \hat{t}_{1T} + \frac{S_\parallel P_{h\perp}^i}{m_h} \hat{t}_{1L}^h + \frac{S_\parallel k_\perp^i}{m_N} \hat{t}_{1L}^\perp \\ & + \frac{(\mathbf{P}_{h\perp} \cdot \mathbf{S}_\perp) P_{h\perp}^i}{m_h^2} \hat{t}_{1T}^{hh} + \frac{(\mathbf{k}_\perp \cdot \mathbf{S}_\perp) k_\perp^i}{m_N^2} \hat{t}_{1T}^{\perp\perp} \\ & + \frac{(\mathbf{k}_\perp \cdot \mathbf{S}_\perp) P_{h\perp}^i - (\mathbf{P}_{h\perp} \cdot \mathbf{S}_\perp) k_\perp^i}{m_N m_h} \hat{t}_{1T}^{\perp h} \\ & + \frac{\epsilon_\perp^{ij} P_{h\perp j}}{m_h} \hat{t}_1^h + \frac{\epsilon_\perp^{ij} k_{\perp j}}{m_N} \hat{t}_1^\perp, \end{aligned}$$

where \mathbf{k}_\perp is the quark transverse momentum and by the vector product of two-dimensional vectors \mathbf{a} and \mathbf{b} we mean the pseudo-scalar quantity $\mathbf{a} \times \mathbf{b} = \epsilon^{ij} a_i b_j = ab \sin(\phi_b - \phi_a)$. All fracture functions depend on the scalar variables $x_B, k_\perp^2, \zeta, P_{h\perp}^2$ and $\mathbf{k}_\perp \cdot \mathbf{P}_{h\perp}$. For the production of a spinless hadron in the TFR one has [1]:

$$(6) \quad \begin{aligned} & \frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\rightarrow\ell(\ell')+h(P)+X}}{dx_B dy d\zeta d^2\mathbf{P}_{h\perp} d\phi_S} = \frac{\alpha_{\text{em}}^2}{Q^2 y} \\ & \times \left\{ [1 + (1-y)^2] \sum_a e_a^2 \left[\tilde{u}_1(x_B, \zeta, P_{h\perp}^2) - S_T \frac{P_{h\perp}}{m_h} \tilde{u}_{1T}^h(x_B, \zeta, P_{h\perp}^2) \sin(\phi_h - \phi_S) \right] \right. \\ & \left. + \lambda y (2-y) \sum_a e_a^2 \left[S_L \tilde{l}_{1L}(x_B, \zeta, P_{h\perp}^2) + S_T \frac{P_{h\perp}}{m_h} \tilde{l}_{1T}^h(x_B, \zeta, P_{h\perp}^2) \cos(\phi_h - \phi_S) \right] \right\}, \end{aligned}$$

where the \mathbf{k}_\perp -integrated fracture functions are given as

$$\begin{aligned} \tilde{u}_1(x_B, \zeta, P_{h\perp}^2) &= \int d^2\mathbf{k}_\perp \hat{u}_1, & \tilde{u}_{1T}^h(x_B, \zeta, P_{h\perp}^2) &= \int d^2\mathbf{k}_\perp \left(\hat{u}_{1T}^h + \frac{m_h}{m_N} \frac{\mathbf{k}_\perp \cdot \mathbf{P}_{h\perp}}{P_{h\perp}^2} \hat{u}_{1T}^\perp \right), \\ (7) \quad \tilde{l}_{1L}(x_B, \zeta, P_{h\perp}^2) &= \int d^2\mathbf{k}_\perp \hat{l}_{1L}, & \tilde{l}_{1T}^h(x_B, \zeta, P_{h\perp}^2) &= \int d^2\mathbf{k}_\perp \left(\hat{l}_{1T}^h + \frac{m_h}{m_N} \frac{\mathbf{k}_\perp \cdot \mathbf{P}_{h\perp}}{P_{h\perp}^2} \hat{l}_T^\perp \right). \end{aligned}$$

We see that a single hadron production in the TFR of SIDIS does not provide access to all fracture functions. At LO the cross-section, with unpolarized leptons, contains only the Sivers-like single spin azimuthal asymmetry.

2. – Double hadron leptonproduction (DSIDIS)

In order to have access to all fracture functions one has to "measure" the scattered quark transverse polarization, for example exploiting the Collins effect [6] – the azimuthal correlation of the fragmenting quark transverse polarization, \mathbf{s}'_T , with the produced hadron transverse momentum, \mathbf{p}_\perp :

$$(8) \quad D(z, \mathbf{p}_\perp) = D_1(z, p_\perp^2) + \frac{\mathbf{p}_\perp \times \mathbf{s}'_T}{m_h} H_1^\perp(z, p_\perp^2),$$

where $s'_T = D_{nn}(y) s_T$ and $\phi_{s'} = \pi - \phi_s$ with $D_{nn}(y) = [2(1-y)]/[1 + (1-y)^2]$.

Let us consider a double hadron production process (DSIDIS)

$$(9) \quad l(\ell) + N(P) \rightarrow l(\ell') + h_1(P_1) + h_2(P_2) + X$$

with (unpolarized) hadron 1 produced in the CFR ($x_{F1} > 0$) and hadron 2 in the TFR ($x_{F2} < 0$), see Fig. 1. For hadron h_1 we will use the ordinary scaled variable $z_1 = P_1^+/k'^+ \simeq P \cdot P_1 / P \cdot q$ and its transverse momentum $\mathbf{P}_{1\perp}$ (with magnitude $P_{1\perp}$ and azimuthal angle ϕ_1) and for hadron h_2 the variables $\zeta_2 = P_2^-/P^- \simeq E_2/E$ and $\mathbf{P}_{2\perp}$ ($P_{2\perp}$ and ϕ_2).

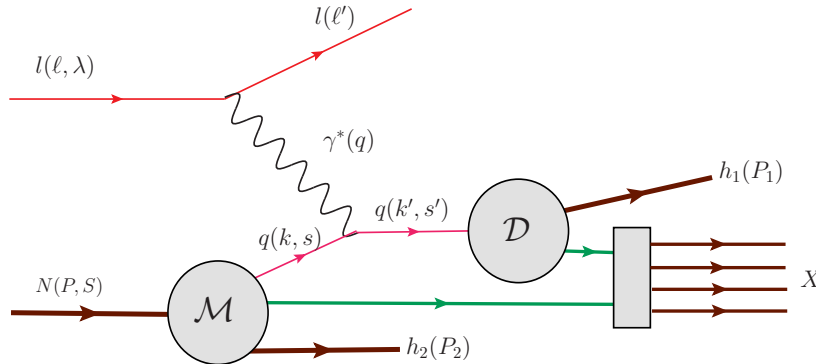


Fig. 1. – DSIDIS description in factorized approach at LO.

In this case the LO expression for the DSIDIS cross-section includes all fracture functions:

$$(10) \quad \frac{d\sigma^{l(\ell,\lambda)+N(P,S)\rightarrow l(\ell')+h_1(P_1)+h_2(P_2)+X}}{dx dy dz_1 d\zeta_2 d^2\mathbf{P}_{1\perp} d^2\mathbf{P}_{2\perp} d\phi_S} = \frac{\alpha^2 x_B}{Q^4 y} [1 + (1-y)^2] \times \\ \left(\mathcal{M}_{h_2}^{[\gamma^-]} \otimes D_{1q}^{h_1} + \lambda D_{ll}(y) \mathcal{M}_{h_2}^{[\gamma^- \gamma_5]} \otimes D_q^{h_1} + \mathcal{M}_{h_2}^{[i\sigma^{i-}\gamma_5]} \otimes \frac{\mathbf{p}_\perp \times \mathbf{s}'_T}{m_{h_1}} H_{1q}^{\perp h_1} \right) = \\ \frac{\alpha^2 x_B}{Q^4 y} [1 + (1-y)^2] (\sigma_{UU} + S_\parallel \sigma_{UL} + S_\perp \sigma_{UT} + \lambda D_{ll} \sigma_{LU} + \lambda S_\parallel D_{ll} \sigma_{LL} + \lambda S_\perp D_{ll} \sigma_{LT}) ,$$

where $D_{ll}(y) = y(2-y)/(1+(1-y)^2)$.

3. – DSIDIS cross-section integrated over $\mathbf{P}_{2\perp}$

If we integrate the fracture matrix over $\mathbf{P}_{2\perp}$ we are left with eight k_\perp -dependent fracture functions:

$$(11) \quad \int d^2\mathbf{P}_{2\perp} \mathcal{M}^{[\gamma^-]} = u_1 + \frac{\mathbf{k}_\perp \times \mathbf{S}_\perp}{m_N} u_{1T}^\perp , \\ (12) \quad \int d^2\mathbf{P}_{2\perp} \mathcal{M}^{[\gamma^- \gamma_5]} = S_\parallel l_{1L} + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{m_N} l_{1T} , \\ \int d^2\mathbf{P}_{2\perp} \mathcal{M}^{[i\sigma^{i-}\gamma_5]} = S_\perp^\perp t_{1T} + \frac{S_\parallel k_\perp^i}{m_N} t_{1L}^\perp + \frac{k_\perp^i (\mathbf{k}_\perp \cdot \mathbf{S}_\perp)}{m_N^2} t_{1T}^\perp + \frac{\epsilon_{ij} k_{\perp j}}{m_N} t_1^\perp \\ (13) \quad = S_\perp^\perp t_1 + \frac{S_\parallel k_\perp^i}{m_N} t_{1L}^\perp + \frac{(k_\perp^i k_\perp^j - \frac{1}{2} \mathbf{k}_\perp^2 \delta_{ij}) S_\perp^j}{m_N^2} t_{1T}^\perp + \frac{\epsilon_{ij} k_{\perp j}}{m_N} t_1^\perp ,$$

where $t_1 \equiv t_{1T} + (\mathbf{k}_\perp^2/2m_N^2) t_{1T}^\perp$. We have removed the hat to denote the $\mathbf{P}_{2\perp}$ -integrated fracture functions, for example:

$$(14) \quad t_1(x_B, \mathbf{k}_\perp^2, \zeta) = \int d^2\mathbf{P}_{2\perp} \left\{ \hat{t}_{1T} + \frac{\mathbf{k}_\perp^2}{2m_N^2} \hat{t}_{1T}^\perp + \frac{\mathbf{P}_{2\perp}^2}{2m_2^2} \hat{t}_{1T}^{hh} \right\} .$$

The complete expression for other seven $\mathbf{P}_{2\perp}$ -integrated fracture functions are presented in Ref. [2].

These $\mathbf{P}_{2\perp}$ -integrated fracture functions are perfectly analogous to those describing single-hadron leptonproduction in the CFR [3], the correspondence being: Fracture Functions \Rightarrow Distribution Functions. Thus we can use the procedure of Ref. [3] to obtain the final expression of the cross section as

$$\frac{d\sigma}{dx_B dy dz_1 d\zeta_2 d\phi_1 dP_{T1}^2 d\phi_S} = \frac{\alpha_{em}^2}{x_B y Q^2} \left\{ \left(1 - y + \frac{y^2}{2} \right) \mathcal{F}_{UU,T} + (1-y) \cos 2\phi_1 \mathcal{F}_{UU}^{\cos 2\phi_1} \right. \\ + S_\parallel (1-y) \sin 2\phi_1 \mathcal{F}_{UL}^{\sin 2\phi_1} + S_\parallel \lambda y \left(1 - \frac{y}{2} \right) \mathcal{F}_{LL} \\ + S_T \left(1 - y + \frac{y^2}{2} \right) \sin(\phi_1 - \phi_S) \mathcal{F}_{UT}^{\sin(\phi_1 - \phi_S)} \\ \left. + S_T (1-y) \sin(\phi_1 + \phi_S) \mathcal{F}_{UT}^{\sin(\phi_1 + \phi_S)} + S_T (1-y) \sin(3\phi_1 - \phi_S) \mathcal{F}_{UT}^{\sin(3\phi_1 - \phi_S)} \right\}$$

$$(15) \quad + S_T \lambda y \left(1 - \frac{y}{2} \right) \cos(\phi_1 - \phi_S) \mathcal{F}_{LT}^{\cos(\phi_1 - \phi_S)} \Big\}$$

where the structure functions are given by the same convolutions as in [3] with the replacement of the TMDs with the $\mathbf{P}_{2\perp}$ -integrated fracture and fragmentation functions: $f \rightarrow u, g \rightarrow l$ and $h \rightarrow t$.

4. – DSIDIS cross-section integrated over \mathbf{P}_{T1}

If one integrates the DSIDIS cross-section over $\mathbf{P}_{1\perp}$ and the quark transverse momentum only one fragmentation function, D_1 , survives, which couples to the unpolarized and the longitudinally polarized \mathbf{k}_\perp -integrated fracture functions:

$$(16) \quad \int d^2 \mathbf{k}_\perp \mathcal{M}^{[\gamma^-]} = \tilde{u}_1(x_B, \zeta_2, P_{2\perp}^2) + \frac{\mathbf{P}_{2\perp} \times \mathbf{S}_T}{m_2} \tilde{u}_{1T}^h(x_B, \zeta_2, P_{2\perp}^2),$$

$$(17) \quad \int d^2 \mathbf{k}_\perp \mathcal{M}^{[\gamma^- \gamma_S]} = S_\parallel \tilde{l}_{1L}(x_B, \zeta_2, P_{2\perp}^2) + \frac{\mathbf{P}_{2\perp} \cdot \mathbf{S}_T}{m_2} \tilde{l}_{1T}^h(x_B, \zeta_2, P_{2\perp}^2),$$

where the fracture functions with a tilde (which means integration over the quark transverse momentum) are as in Eqs. (7).

The final result for the cross section is [2]

$$(18) \quad \begin{aligned} \frac{d\sigma}{dx_B dy dz_1 d\zeta_2 d\phi_2 dP_{2\perp}^2 d\phi_S} &= \frac{\alpha_{\text{em}}^2}{y Q^2} \left\{ \left(1 - y + \frac{y^2}{2} \right) \right. \\ &\times \sum_a e_a^2 \left[\tilde{u}_1(x_B, \zeta_2, P_{2\perp}^2) - S_T \frac{P_{2\perp}}{m_2} \tilde{u}_{1T}^h(x_B, \zeta_2, P_{2\perp}^2) \sin(\phi_2 - \phi_S) \right] \\ &+ \lambda y \left(1 - \frac{y}{2} \right) \sum_a e_a^2 \left[S_\parallel \tilde{l}_{1L}(x_B, \zeta_2, P_{2\perp}^2) \right. \\ &\left. \left. + S_T \frac{P_{2\perp}}{m_2} \tilde{l}_{1T}^h(x_B, \zeta_2, P_{2\perp}^2) \cos(\phi_2 - \phi_S) \right] \right\} D_1(z). \end{aligned}$$

As in the case of single-hadron production [1], there is a Sivers-type modulation $\sin(\phi_2 - \phi_S)$, but no Collins-type effect.

5. – Examples of unintegrated cross-sections: beam spin asymmetry

We show here explicit expressions only for σ_{UU} and σ_{LU} ⁽¹⁾

$$(19) \quad \begin{aligned} \sigma_{UU} &= F_0^{\hat{u} \cdot D_1} - D_{nn} \left[\frac{P_{1\perp}^2}{m_1 m_N} F_{kp1}^{\hat{t}^\perp \cdot H_1^\perp} \cos(2\phi_1) + \frac{P_{1\perp} P_{2\perp}}{m_1 m_2} F_{p1}^{\hat{t}^h \cdot H_1^\perp} \cos(\phi_1 + \phi_2) \right. \\ &\left. + \left(\frac{P_{2\perp}^2}{m_1 m_N} F_{kp2}^{\hat{t}^\perp \cdot H_1^\perp} + \frac{P_{2\perp}^2}{m_1 m_2} F_{p2}^{\hat{t}^h \cdot H_1^\perp} \right) \cos(2\phi_2) \right]. \end{aligned}$$

⁽¹⁾ Expressions for other terms are available in [7].

$$(20) \quad \sigma_{LU} = -\frac{P_{1\perp}P_{2\perp}}{m_2m_N} F_{k1}^{\hat{t}^{\perp h} \cdot D_1} \sin(\phi_1 - \phi_2),$$

where the structure functions F_{k1}^{\dots} are specific convolutions [7, 8] of fracture and fragmentation functions depending on $x, z_1, \zeta_2, P_{1\perp}^2, P_{2\perp}^2, \mathbf{P}_{1\perp} \cdot \mathbf{P}_{2\perp}$.

We notice the presence of terms similar to the Boer-Mulders term appearing in the usual CFR of SIDIS. What is new in DSIDIS is the LO beam spin SSA, absent in the CFR of SIDIS. We further notice that the DSIDIS structure functions may depend in principle on the relative azimuthal angle of the two hadrons, due to presence of the last term among their arguments: $\mathbf{P}_{1\perp} \cdot \mathbf{P}_{2\perp} = P_{1\perp}P_{2\perp} \cos(\Delta\phi)$ with $\Delta\phi = \phi_1 - \phi_2$. This term arise from $\mathbf{k}_\perp \cdot \mathbf{P}_\perp$ correlations in STMD fracture functions and can generate a long range correlation between hadrons produced in CFR and TFR. In practice it is convenient to chose as independent azimuthal angles $\Delta\phi$ and ϕ_2 .

Let us finally consider the beam spin asymmetry defined as

$$(21) \quad A_{LU}(x, z_1, \zeta_2, P_{1\perp}^2, P_{2\perp}^2, \Delta\phi) = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = \frac{-\frac{P_{1\perp}P_{2\perp}}{m_2m_N} F_{k1}^{\hat{t}^{\perp h} \cdot D_1} \sin(\Delta\phi)}{F_0^{\hat{u} \cdot D_1}}.$$

If one keeps only the linear terms of the corresponding fracture function expansion in series of $\mathbf{P}_{1\perp} \cdot \mathbf{P}_{2\perp}$ one obtains the following azimuthal dependence of DSIDIS beam spin asymmetry:

$$(22) \quad A_{LU}(x, z_1, \zeta_2, P_{1\perp}^2, P_{2\perp}^2) = a_1 \sin(\Delta\phi) + a_2 \sin(2\Delta\phi)$$

with the amplitudes a_1, a_2 independent of azimuthal angles.

We stress that the ideal opportunities to test the predictions of the present approach to DSIDIS, would be the future JLab 12 upgrade, in progress, and the EIC facilities, in the planning phase.

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